Rigorous Formulation for Fields and Currents in Superconducting Microwave Transmission Lines

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Abstract—**A direct approach is described for obtaining current distributions, power handling capabilities, and propagation characteristics of high T_c superconductor microwave lines. A rigorous formulation based on coupling a full-wave electromagnetic model with London's equations and the two fluid model for superconducting materials is suggested. The finite-difference scheme is employed to obtain a simplified solution. Calculated results showing current distributions and quality factor of a superconducting microstrip line are presented.**

I. Introduction

INCORPORATING high T_c superconducting (HTS) materials in planar transmission lines is very promising for high speed digital [1], and high frequency analog [2] applications. Before full exploitation of the HTS's, their current and power handling capacities need to be assessed. Knowing the critical current density of the material does not directly lead to the maximum current that can be carried by the transmission line since the current is not uniformly distributed in the cross-section. Superconducting microwave devices were theoretically investigated to obtain their propagation characteristics [3], [4]. In this letter, London's equations and the two-fluid model are used to investigate the current distributions on HTS transmission lines. They are coupled with Maxwell's equations to develop a rigorous full-wave model. The total current carried by an HTS microstrip line without exceeding the critical current density of the material as well as the total power can be calculated. The losses due to normal electrons and the Q-factor are directly obtained. Initial calculations have been performed. Preliminary results, aiming at demonstrating the potential of this approach, are presented. The flexibility of this technique is appreciated by noticing that it can easily be modified to incorporate complex issues of HTS materials, including the nonlinearity of the parameters and the anisotropy.

II. Currents and Fields in Superconducting Lines

HTS materials (e.g., $Tl_2Ba_2Ca_2Cu_3O_{10}$) are considered type-II superconductors, which usually considered as having a coherence length much smaller than the penetration depth. The current carrying mechanism in Type-II superconductors is considered a local phenomenon, which means that the current density at any point may be described by the local field at the same point. London's equations and the two-fluid model can be used to macroscopically predict the relation between the local field and the current density. The conducting electrons in a superconductor are divided into two categories: superconducting electrons, known as Cooper pairs or electron pairs, and normal electrons [5]. The electron pair transport is assumed to be collision-free, while the normal electron transport is governed by the momentum conservation equation. London's equations, which relate the local electromagnetic fields and the superconducting current density $\boldsymbol{J}_s$ and the penetration depth λ, are as follows.

$$\frac{\partial \boldsymbol{J}_s}{\partial t} = \frac{1}{\mu_0 \lambda^2} \boldsymbol{E}, \tag{1}$$

$$\nabla \times \boldsymbol{J}_s = \frac{-1}{\lambda^2} \boldsymbol{H}. \tag{2}$$

Assuming that both London's equations and the two-fluid model are valid for the frequency range of interest, the full-wave analysis can be derived as follows. The starting point is Maxwell's equation,

$$\nabla \times \boldsymbol{H} = j\omega\epsilon \boldsymbol{E} + \boldsymbol{J}_s + \boldsymbol{J}_n, \tag{3}$$

where $\boldsymbol{J}_n$ is the normal current density, and time dependence in the form $e^{j\omega t}$ is understood. Substituting (1) and writing $\boldsymbol{J}_n = \sigma_n \boldsymbol{E}$, in (3) results in

$$\nabla \times \boldsymbol{H} = \xi \boldsymbol{J}_s, \tag{4}$$

where

$$\xi = -\omega^2 \mu_0 \epsilon \lambda^2 + j\omega\mu_0\sigma_n\lambda^2 + 1. \tag{5}$$

Using the magnetic vector potential $\boldsymbol{A}$ to replace $\boldsymbol{H}$ in (2), via the definition $\boldsymbol{H} = (\nabla \times \boldsymbol{A})/\mu_0$, one obtains

$$\boldsymbol{J}_s = \frac{-1}{\mu_0 \lambda^2} \boldsymbol{A} + \nabla\Psi, \tag{6}$$

where Ψ is a scalar function to be determined from the boundary conditions. Using (6) to eliminate $\boldsymbol{J}_s$ from (4) results in the following for $\boldsymbol{A}$:

$$\nabla^2 \boldsymbol{A} - \frac{\xi}{\lambda^2} \boldsymbol{A} = -\mu_0 \xi \nabla\Psi, \tag{7}$$

where Coulomb's gauge ($\nabla . \boldsymbol{A} = 0$) was used.

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Equation (7) is a distorted form of a nonhomogeneous vector wave equation. We labeled it distorted due to the term "ξ/λ^2," which is different from the conventional wave number. It is used in the different media to describe the magnetic vector potential distribution by supplying the appropriate values of ϵ, λ and σ_n, assuming that $\mu = \mu_0$ everywhere. In the air and dielectric regions of the transmission line, (7) reduces to the conventional homogeneous vector wave equation.

The variable $\nabla\Psi$ appearing in (6) and subsequent ones, is directly related to the superconducting current distributions. The equation governing this function can be derived by substituting (6) in Maxwell's curl equation as follows.

$$\nabla \times \boldsymbol{H} = j\omega\epsilon \boldsymbol{E} - \frac{1}{\mu_0 \lambda^2} \boldsymbol{A} + \nabla\Psi + \boldsymbol{J}_n. \tag{8}$$

Taking the divergence of (8), using Coulomb's gauge and $\nabla \cdot \boldsymbol{E} = \rho/\epsilon$ yields

$$\nabla^2\Psi = -j\omega(\rho - \rho_n), \tag{9}$$

where ρ and ρ_n are the total and normal charge densities respectively. Equation (9) is the basic definition for the complementary part of the superconductor current density. The relation between ρ and ρ_n is obtained from the two fluid model.

To gain a physical insight into and assess the importance of $\nabla\Psi$, assume $\rho_n \ll \rho$ and quasi-TEM wave propagation with dependence $e^{-j\beta z}$. Therefore, the current density flows along the direction of the wave propagation. In this case, only the z-component of $\nabla\Psi$ is needed. Hence, $\nabla^2\Psi$ simplifies to

$$\nabla^2\Psi \approx \frac{\partial}{\partial z} \nabla\Psi . \boldsymbol{a}_z = -j\beta \nabla\Psi . \boldsymbol{a}_z, \tag{10}$$

which leads to

$$\nabla\Psi . \boldsymbol{a}_z = \frac{\omega}{\beta} \rho = v_{\text{ph}} \rho, \tag{11}$$

where v_{ph} is the phase velocity of the propagating wave. This shows that $\nabla\Psi$ is a two-dimensional function related to the surface current density on the superconductor surface since ρ is zero everywhere except near the surface. Due to the strong singularity exhibited by ρ at the sharp edges, this function evidently plays an important role in shaping all the current and field distributions. Alsop *et al.* derived expressions for the current distributions in superconductors at dc. However, $\nabla\Psi$ was considered as a constant in their analysis [6]. This results in inaccurate results in both current and field distributions in microwave structures. Moreover, this leads to nonphysical results in the form of current distributions in a superconducting transmission line that are independent of the dielectric constants of the substrates. It is worth mentioning that sometimes this function could be approximated by a constant as, for example, the case of a very wide microstrip line over a thin substrate and a thick superconductor (i.e., $W \gg d$ and $t \gg \lambda$). Yet, the significance of this approximation is that the microstrip line is approximated as a parallel-plate transmission line, which is a trivial one-dimensional case.

III. Simplified Solution and Results

A simplified, yet accurate, solution for the previous derived equations can be obtained knowing that for typical HTS materials, $\xi \approx 1$ for frequencies up to 10^{12} Hz. Moreover, the practical microwave lines are normally operated in the quasi-TEM mode. Hence, (7) reduces to

$$\nabla_t^2 A_z - \frac{1}{\lambda^2} A_z = -\mu_0 \nabla\Psi . \boldsymbol{a}_z, \tag{12a}$$

and

$$\nabla_t^2 A_z = 0, \tag{12b}$$

where ∇_t^2 is the Laplacian operator in the transverse plane (i.e., xy plane), (12a) is used inside the superconductor and (12b) is used in the air and dielectric regions. The current flows only axially in the z direction. These two equations are discretized over the transverse plane using the finite-difference scheme and solved for A_z. Once A_z is obtained, the current and field distributions are readily calculated. The normal current density and the losses are calculated using the two fluid model and the perturbation approach.

To demonstrate the potential of this approach, it was applied to the HTS microstrip line structure shown in Fig. 1; with $2W = 500$ μm, d = 425 μm and $t = 1$ μm. The substrate is made of a loss-less material, which has a relative dielectric constant of 23. The superconductor is characterized by $T_c = 100$ K, the penetration depth at $T = 0$ K, is $\lambda(0) = 0.18$ μm, the density of electrons is 10^{21} cm^{-3}, and the conductivity of normal electrons $\sigma_n \approx 10^4$ S/cm at T_c. A nonuniform two dimensional mesh is used. The simulation region is extended to 12W in the x direction, and to $5d$ in the y direction. Magnetic walls are used to terminate the open boundaries. Exploiting the symmetry about the x axis, only one half of the structure is simulated.

Fig. 2 shows the current density distributions inside the HTS strip as functions of x at constant y planes, at $T = 50$ K. The total current carried by the strip is normalized to 100 mA. It is shown that the current is mainly carried by the superconductor surface adjacent to the dielectric substrate. The current decreases with the increase of y, and it slightly increases as y approaches t as shown in the insert in Fig. 2. This phenomenon is due to the high dielectric constant of the material used in the substrate, which represents one of the typical materials currently used in industry. The unloaded quality factor Q of the strip is shown in Fig. 3. A monotonous decrease of the Q with frequency and temperature is shown as expected.

IV. Conclusion

A rigorous formulation for HTS microwave lines is presented. It couples a full-wave electromagnetic model with both London's equations and the two-fluid model. An approximate solution for the derived equation is developed using the finite difference scheme. The potential of this technique is demonstrated by investigating current density distributions and Q of a superconductor microstrip line. This technique can be applied to other planar transmission lines with superconducting materials.

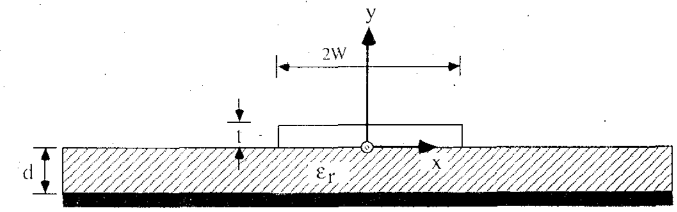

Fig. 1. Simulated microstrip line.

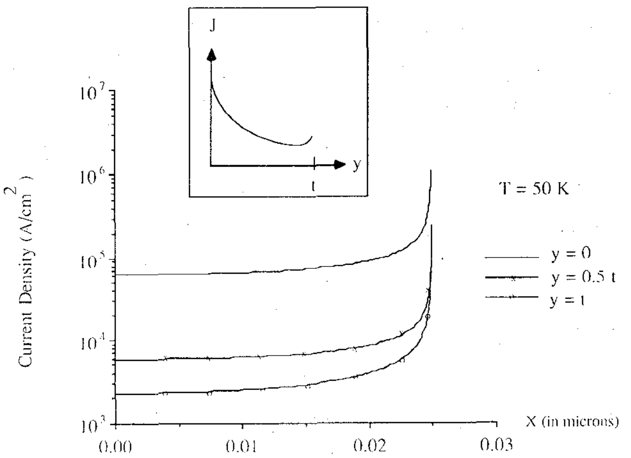

Fig. 2. Current density distributions inside the superconducting strip as functions of x at constant y planes.

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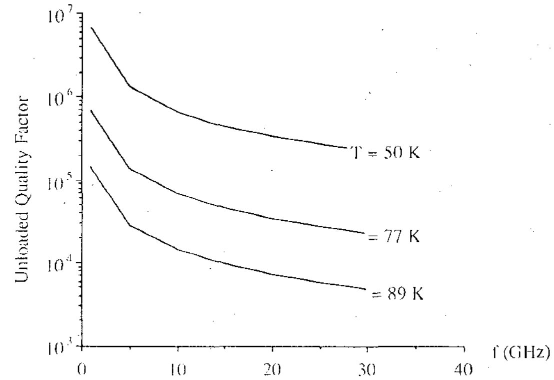

Fig. 3. Unloaded Q as a function of frequency at different temperatures.

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